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FEM PREDICTIONS FOR TWO-PHASE FLOW IN A VERTICAL PIPE WITHOUT THE USE OF ANY EXTERNAL CORRELATIONS

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ABSTRACT

A mathematical model is developed for the description of the thermohydraulics of the two-phase flow phenomenon in a vertical pipe. Using an additional momentum equation for the slip velocity, it is shown that the computation of slip and pressure drop from the model equations is possible without the use of any external correlations. The finite element method is used to solve the governing equations. The predictions for a steam-water two-phase flow in vertical upflow with constant wall heat flux agree well with experimental results and with widely used correlations.

KEY WORDS Two-phase flow Pressure drop Void fraction Slip ratio Steady flow Vertical upflow

NOMENCLATURE

B_0	boiling number,
f	friction factor,
g	gravity constant, m/sec^2
G	mass flow velocity, kg/m^2 sec,
h	specific enthalpy, J/kg,
p	system pressure, N/m^2
$\dot{q}$	heat flux density, W/m^3,
r	radius of the tube, m,
S	slip ratio,
T	temperature, °C,
v	specific volume, m^3/kg,
w	velocity, m/sec,
x	mass fraction,
z	fluid space coordinate, m,
ρ	density, kg/m^3,
Φ	inclination angle (fluid flow),
Φ_{fo}^2	two-phase friction multiplier,
ε	void fraction,

Indices

1—liquid phase; 2—gaseous phase;
21—phase difference

INTRODUCTION

Two-phase flow takes place in a wide range of industrial plants, boilers, nuclear reactors, refrigeration systems etc. In these flow systems, it is important to be able to predict the pressure drops. Their evaluation is necessary in the design of the fluid systems, in functional verifications and in safety analysis of a great variety of components and systems. There are quite a number of rather extensive models for the description of the working fluid. Many different approximate models for the momentum equation have been proposed. These models differ both in simplifications adopted to simulate interphase interactions and especially in the choice of the parameters considered for expressing the flow evolution. Separated flow models involve the evaluation of the phase interaction which is complicated by mathematical difficulty. For example,

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refer to Bergles *et al.*[1], Cumo and Naviglio[2] or Wallis[3]. These models use external correlations for their predictions or in describing the interfacial interactions. The model proposed herein gives results of an extended mathematical description of two-phase flow fluid behaviour stating the possibility of a consistent characterization of the most relevant two-phase flow parameters, i.e. slip and pressure drop, without the use of any external correlations. The finite element method is used to solve the one-dimensional steady state model for vertical upflow of water during forced convection boiling with constant wall heat flux in round tubes. Comparisons are made with the available experimental data of Miropolskii[4], Christensen[5] and with the well known correlations of Martinelli-Nelson[6]. The predicted results are in reasonably good agreement with the experimental data and correlations.

MATHEMATICAL MODEL EQUATIONS OF TWO-PHASE FLOW SYSTEMS

The description of the state of a flowing medium is done by using the fundamental conservation laws for mass, energy and momentum leading to a set of partial differential balance equations. If the medium investigated is in the form of a multi-component mixture, the balance equations have to be written for each component of the mixture. Moreover, the balance equations for each component must account for the mass, momentum and energy exchanges between the different components. In the present work it is assumed that the investigated system is in thermodynamic equilibrium at each instant. A single component steady flow steam generator system is considered here with vertical upflow of water. The one-dimensional analysis in this work closely follows the theoretical work of Schittke[7], Soo[8,9] and Sha and Soo[10].

Figure 1 shows the configuration used for the simulation of the actual steam generator system. The actual steam generator is approximated by a single tube with a circular cross-section. The fluid (water) flows vertically upwards through the tube which is heated with constant wall heat flux. Depending on the state of water phases, there are three different regions in the steam generator; two of them (the single-phase sections, economizer and superheater) are treated mathematically equivalent. In the evaporator section, influenced by heating, water undergoes a phase change leading to a two-phase flow system as both phases are simultaneously present throughout the evaporator section. Both phases are assumed to be separate. The results obtained

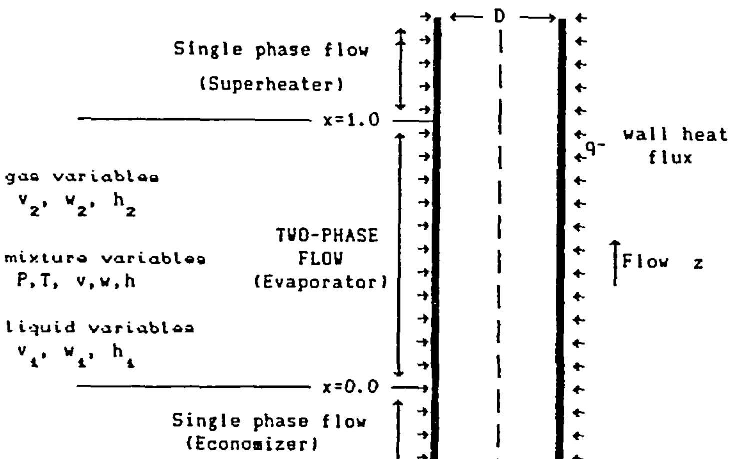

Figure 1 Flow geometry (one-dimensional flow)

are valid for any flow regime, because the model derivation is independent of this constraint. Principal flow direction of the fluid is termed as z.

Mass conservation

The density of the mixture is given by[8]:

$$\rho = \rho^{(1)} + \rho^{(2)} \tag{1}$$

$\rho^{(1)}$ and $\rho^{(2)}$ are the partial densities (not equivalent to the true densities ρ_1 for the liquid and ρ_2 for the gaseous phase).

Mass fraction or quality is defined as:

$$x = \frac{\rho^{(2)}}{\rho} \tag{2}$$

Using specific volumes v_1 and v_2 of the liquid and gaseous phases rather than densities, the mean specific volume of the mixture is defined as:

$$v = v_1 + x v_{21} \tag{3}$$

where

$$v_{21} = v_2 - v_1 \tag{4}$$

The mean mixture velocity is derived by summing the mass flow densities of the two-phases, with phase velocities w_1 and w_2 respectively[8]:

$$\rho w = \rho^{(1)} w_1 + \rho^{(2)} w_2 \tag{5}$$

i.e.

$$w = w_1 + x w_{21} \tag{6}$$

where

$$w_{21} = w_2 - w_1 \tag{7}$$

w_{21} is the phase difference velocity, which is labelled as 'slip velocity'. The mass balance equation of the mixture follows from continuity and is written as:

$$w \frac{\mathrm{d}v}{\mathrm{d}z} = v \frac{\mathrm{d}w}{\mathrm{d}z} \tag{8}$$

Energy conservation

Accordingly the energy equation of the mixture is:

$$w \frac{\mathrm{d}h}{\mathrm{d}z} = v \dot{q} \tag{9}$$

with specific enthalpy h, defined by the respective phase enthalpies h_1 and h_2 is given by:

$$h = h_1 + x(h_2 - h_1) \tag{10}$$

$\dot{q}$ is the heat flux density induced on the fluid flow. No energy balance equation for any of the two phases is used because in this context the actual amount of energy transferred from one phase to the other is of no importance.

Momentum conservation

For a Newtonian fluid the total momentum gradient is equivalent to the sum of the driving forces. Momentum balance equation for mixture is written as:

$$w\frac{dw}{dz} = -v\frac{dp}{dz} - g\sin\phi - fw^2 \tag{11}$$

p is the pressure of the mixture (which is assumed to be identical for both phases for any value of z). The driving forces are pressure head, gravity head and friction influencing the flow system. Usually, the friction term is evaluated using external empirical correlations by various investigators, e.g., Collier[11]. In the present analysis no external correlations are used and the friction term is evaluated using the momentum balance equations of individual phases.

Momentum balance equations for each of the two phases[8,9] are found to be after some transformations:

$$w_1\frac{dw_1}{dz} = -v_1\frac{dp}{dz} - (w_1 - w)\frac{dw}{dz} - g\sin\phi - f_1 w_1^2 \tag{12}$$

$$w_2\frac{dw_2}{dz} = -v_2\frac{dp}{dz} - (w_2 - w)\frac{dw}{dz} - g\sin\phi - f_2 w_2^2 \tag{13}$$

for the liquid and gaseous phases respectively. The above equations are not used directly for the present model. The balance equation for the slip velocity w_{21} is derived by subtracting (12) from (13) and simplifying, the balance equation for slip velocity becomes:

$$w\frac{dw_{21}}{dz} = -v_{21}\frac{dp}{dz} - w_{21}\frac{dw}{dz} - f_{21}w^2 \tag{14}$$

where the slip velocity friction factor f_{21} is given by:

$$f_{21}w^2 = f_2 w_2^2 - f_1 w_1^2 \tag{15}$$

It is seen that the gravity head source term is not present in the momentum balance equation for the slip velocity. Instead, there is a new source term describing the influence of the mean velocity gradient.

With the knowledge of the mean velocity w, the slip velocity w_{21} and the mass fraction x, the true phase velocities w_1 and w_2 are computed. Thus, slip ratio is calculated using:

$$S = \frac{w_2}{w_1} \tag{16}$$

directly from model equations. For the steady-state behaviour, with the use of an additional momentum balance equation for the slip velocity, it is possible to evaluate slip ratio without any external correlations.

The void fraction is calculated by using[11]:

$$\varepsilon = \frac{1}{1 + S\dfrac{(1-x)}{x}\dfrac{v_1}{v_2}} \tag{17}$$

Similarly, the two-phase friction factor is calculated using an equivalent expression. Assuming that f_1 and f_2 are well known (being the equivalent single phase friction factors depending on the Reynolds number), the summing up of the momentum balance equation for the separate phases according to the definition (6) of the mean velocity, will yield:

$$fw^2 = (1 - x)f_1 w_1^2 + xf_2 w_2^2 \tag{18}$$

The evaluation of f_1 and f_2 depends on the nature of the flow (laminar or turbulent) from the Reynolds number at for the situation at each location of the flow. In majority of the cases turbulent flow will be taking place.

In this way the model is able to perform without any additional use of external correlations for determining slip and friction coefficients. The state variables (such as specific volume, enthalpy and temperature) are dependent upon the system pressure p as shown below:

$$\begin{gathered} v_1 = v_1(p) \qquad v_2 = v_2(p) \\ h_1 = h_1(p) \qquad h_2 = h_2(p) \\ T = T_2(p) \end{gathered} \tag{19}$$

The two-phase friction multiplier ϕ_{fo} commonly in use[11] can be calculated by describing ϕ_{fo}^2 as the ratio of actual two-phase friction pressure drop and a fictitious single-phase frictional pressure drop when the total flow is liquid only with the same mass flow rate as that of two-phase flow.

$$\Phi_{fo}^2 = \frac{v_1}{v}\frac{fw^2}{f_1 w^2} \tag{20}$$

Mathematical model equations for single-phase flow systems

The two cases in the system where single phase flows have to be considered are: (i) sub cooled water in the economizer; (ii) superheated vapour in the superheater. These cases are not considered separately as there is mathematically no significant difference between them.

The balance equations needed here are equivalent to the two-phase balance equations for the mixture mass, energy and momentum conservations. There is no slip velocity equation in single phase flow. The thermodynamic state variables are dependent on the system pressure and temperature:

$$v = v(p, T); \qquad h = h(p, T) \tag{21}$$

Table 1 gives the governing balance equations listing for the single phase and two-phase flow situation.

Table 1 Governing equations for single and two-phase flows

	Single phase	Two-phase
Mass balance	$w\frac{dv}{dz} = v\frac{dw}{dz}$	$w\frac{dv}{dz} = v\frac{dw}{dz}$
Energy balance	$w\frac{dh}{dz} = v\dot{q}$	$w\frac{dh}{dz} = v\dot{q}$
Momentum balance	$w\frac{dw}{dz} = -v\frac{dp}{dz} - g\sin\phi - fw^2$	$w\frac{dw}{dz} = -v\frac{dp}{dz} - g\sin\phi - fw^2$
Slip vel balance	$w\frac{dw_{21}}{dz} = 0$	$w\frac{dw_{21}}{dz} = -v_{21}\frac{dp}{dz} - w_{21}\frac{dw}{dz} - f_{21}w^2$

FINITE ELEMENT FORMULATION

The finite element analysis is used to model this problem to take care of various types of possible boundary conditions along the length of the pipe. Finite element method also facilitates a simpler way for extending the model to 2-dimension/3-dimension.

A one-dimensional linear element with two nodes is used in the following, as shown in *Figure 2*. Galerkin's method is used for the analysis[12].

The unknowns are taken with finite element discretization as:

$$\begin{aligned} \{w\} &= [N]^{w}\{w\} & \{h\} &= [N]^{h}\{h\} \\ \{p\} &= [N]^{p}\{p\} & \{w_{21}\} &= [N]^{w_{21}}\{w_{21}\} \\ \{fw^2\} &= [N]\{fw^2\} & \{f_{21}w^2\} &= [N]\{f_{21}w^2\} \end{aligned} \tag{22}$$

where

$$[N] = [(1-\eta)\eta] \qquad \text{and} \qquad \mathrm{d}z = \Delta v\, \mathrm{d}\eta \tag{23}$$

Then using Galerkin's method:

$$\begin{gathered} \int \left\{ \{w\}\frac{\mathrm{d}\{\mathrm{v}\}}{\mathrm{d}z} - \{v\}\frac{\mathrm{d}\{w\}}{\mathrm{d}z} \right\} [N]^{\mathrm{T}} \Delta z\, \mathrm{d}\eta = 0 \\ \int \left\{ \{w\}\frac{\mathrm{d}\{v\}}{\mathrm{d}z} - \{v\}q \right\} [N] \Delta z\, \mathrm{d}\eta = 0 \\ \int \left\{ \{w\}\frac{\mathrm{d}\{h\}}{\mathrm{d}z} + \{v\}\frac{\mathrm{d}\{p\}}{\mathrm{d}z} + g\sin\phi + \{fw^2\} \right\} [N]^{\mathrm{T}} \Delta z\, \mathrm{d}\eta = 0 \\ \int \left\{ \{w\}\frac{\mathrm{d}\{w_{21}\}}{\mathrm{d}z} + \{v_{21}\}\frac{\mathrm{d}\{p\}}{\mathrm{d}z} + \{w_{21}\}\frac{\mathrm{d}\{w\}}{\mathrm{d}z} + \{f_{21}w^2\} \right\} [N]^{\mathrm{T}} \Delta z\, \mathrm{d}\eta = 0 \end{gathered} \tag{24}$$

Evaluating (24) the stiffness matrix is written as:

$$\underset{(8\times 8)}{[K]} \times \underset{(8\times 1)}{\{\Phi\}} = \underset{(8\times 1)}{\{f\}} \tag{25}$$

where $[K]$ is the elemental stiffness matrix and $\{f\}$ is the elemental force vector as shown below:

$$\begin{bmatrix} K_{11} & 0 & 0 & 0 & K_{15} & 0 & 0 & 0 \\ 0 & K_{22} & 0 & 0 & 0 & K_{26} & 0 & 0 \\ K_{31} & 0 & K_{39} & 0 & K_{35} & 0 & K_{37} & 0 \\ K_{41} & 0 & K_{49} & K_{44} & K_{45} & 0 & K_{47} & K_{48} \\ K_{51} & 0 & 0 & 0 & K_{55} & 0 & 0 & 0 \\ 0 & K_{62} & 0 & 0 & 0 & K_{66} & 0 & 0 \\ K_{71} & 0 & K_{79} & 0 & K_{75} & 0 & K_{77} & 0 \\ K_{81} & 0 & K_{89} & K_{84} & K_{85} & 0 & K_{87} & K_{88} \end{bmatrix} \begin{Bmatrix} w_1 \\ h_1 \\ p_1 \\ w_{21_1} \\ w_2 \\ h_2 \\ p_2 \\ w_{21_2} \end{Bmatrix} = \begin{Bmatrix} 0 \\ f_2 \\ f_3 \\ f_4 \\ 0 \\ f_6 \\ f_7 \\ f_8 \end{Bmatrix}$$

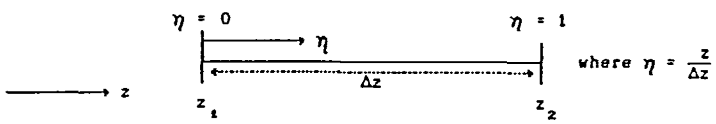

Figure 2 One-dimensional linear element

where:

$$K_{11} = K_{51} = v_2 \qquad K_{15} = K_{55} = -v_1$$

$$K_{22} = K_{31} = K_{35} = K_{44} = -(2w_1 + w_2) \qquad K_{26} = K_{48} = (2w_1 + w_2)$$

$$K_{33} = -(2v_1 + v_2) \qquad K_{37} = (2v_1 + v_2)$$

$$K_{41} = -(2w_{21_1} + w_{21_2}) \qquad K_{43} = -(2v_{21_1} + v_{21_2})$$

$$K_{45} = (2w_{21_1} + w_{21_2}) \qquad K_{47} = (2v_{21_1} + v_{21_2})$$

$$K_{62} = K_{71} = K_{84} = -(w_1 + 2w_2) \qquad K_{66} = K_{75} = K_{88} = (w_1 + 2w_2)$$

$$K_{73} = K_{77} = -(v_1 + 2v_2)$$

$$K_{81} = -(w_{21_1} + w_{21_2}) \qquad K_{83} = -(v_{21_1} + v_{21_2})$$

$$K_{85} = (w_{21_1} + w_{21_2}) \qquad K_{87} = (v_{21_1} + v_{21_2})$$

$$f_2 = q\Delta z(2v_1 + v_2)$$

$$f_3 = \{(3g \sin \phi) + 2(fw^2)_1 + (fw^2)_2\}\Delta z$$

$$f_4 = \{2(f_{21}w^2)_1 + (f_{21}w^2)\}\Delta z$$

$$f_6 = q\Delta z(v_1 + 2v_2)$$

$$f_7 = \{(3g \sin \phi) + (fw^2)_1 + 2(fw^2)_2\}\Delta z$$

$$f_8 = -\{(f_{21}w^2)_1 + 2(f_{21}w^2)\}\Delta z$$

The inlet conditions of the liquid such as pressure, mass flow rate, temperature are known. By solving the elemental matrix, the pressure, mixture velocity, slip velocity and enthalpy are evaluated at the second node. Knowing the conditions at the 2nd node (which is the first node of second element), the pressure, velocity, enthalpy and slip velocity at node 3 is evaluated in a similar manner. In this way we march until the end of the pipe is reached.

The elemental stiffness and force matrices involve the flow and physical parameters (which are dependent on pressure at each location) at node 2 of the same element, which is not known *a priori*. Hence, we assume the values of physical parameters and velocities to be the same in both the nodes and start iteration. Using the calculated values at node 2 during first iteration, next iteration is carried out. The iterations are stopped when the values converge within an accuracy of 0.0001.

NUMERICAL EVALUATION

A computer program is developed in Fortran according to the methodology described. The program takes care of both single and two-phase flow situations. The method is applied to the vertical upflow of steam-water in round tubes. Results are obtained for different combinations of pressures, mass flow rates, wall heat flux and diameters as shown in *Table 2*. The size of each element is taken as 0.001 m. Number of elements depends upon the length of the pipe.

Table 2 Various combinations of flow situations

Pressure (MPa)	4.14	6.89	8.27	10.0	20.0
Mass flux (kg/m^2 sec)	929.0	1223.0	1332.0	1460.0	2664.0
Heat flux (kw/m^2)	100	150	200	355	2689.0
Diameter (mm)	5.0	9.0	10.16	12.5	–

RESULTS AND DISCUSSION

The results obtained from the above analysis using the finite element method for the solution are presented and compared with some of the available data from experiments and also with the well known correlations.

Figure 3 shows the plot of two-phase flow friction multiplier as a function of quality for two pressures 20 and 10 MPa, with mass flow velocities 1332 and 1500 kg/m^2 sec and wall heat flux of 355 and 150 kw/m^2 (Koehler and Kastner[13]). For the sake of comparison, the values were calculated based on homogeneous model[3], Martinelli-Nelson (M–N)[6], Levy[14] and Thom[15] correlations along with experimental data[13] are also plotted. It is observed that the present model predicts the two-phase friction multiplier which lies close to the M–N correlation and lies within the range of other correlations and experimental points. The trends of the predicted Φ^2_{fo} follows that of M–N correlation.

The effect of pressure on the two-phase friction factor is shown in *Figure 4*. The results are in good agreement with M–N correlation[6] and also with the experimental data (for 20 MPa)[13].

The effect of mass velocity upon the two-phase flow friction multiplier, Φ^2_{fo} at a pressure 6.89 MPa is shown in *Figure 5*. Diameter of the pipe is 5 mm. At a fixed value of x, a decrease in Φ^2_{fo} with increased mass velocity is observed. The predictions are compared with experimental results of Muscettola[16]. The M–N[6] and homogeneous models effectively bracket the experimental and present model results. Values of Thom[15] are also shown.

The slip ratio depends on the parameter, v, defined as:

$$\frac{1}{v} = \frac{q}{G h_{21} f_2} = \frac{B_0}{f_2} \tag{26}$$

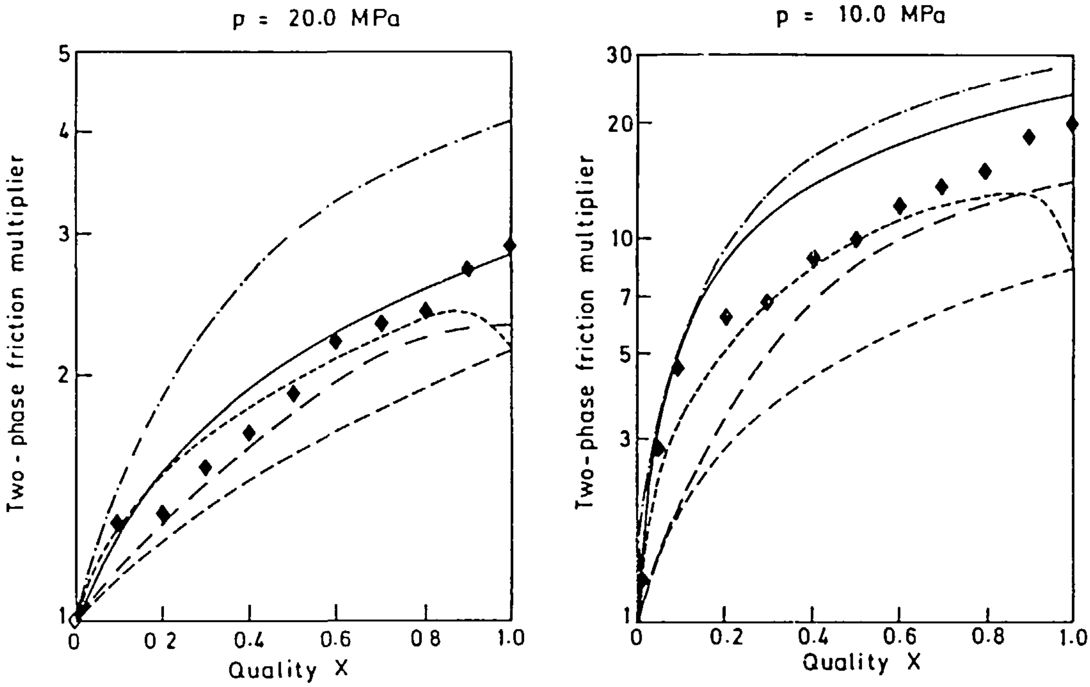

Figure 3 Two-phase friction multiplier variation with quality at 20.0 and 10.0 MPa pressures. ———, present model; -----, Martinelli-Nelson[6]; ♦♦♦♦♦, experimental[13]; — — —, Thom[5]; —·—·—, Levy[14]

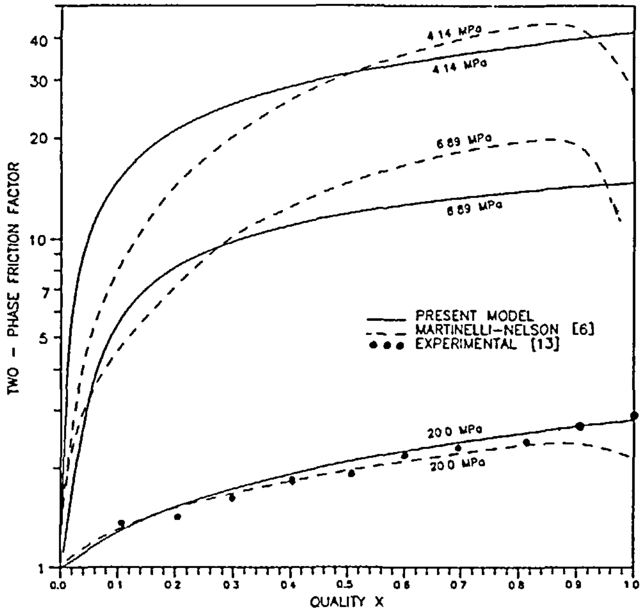

Figure 4 Two-phase friction multiplier variation with quality at different pressures

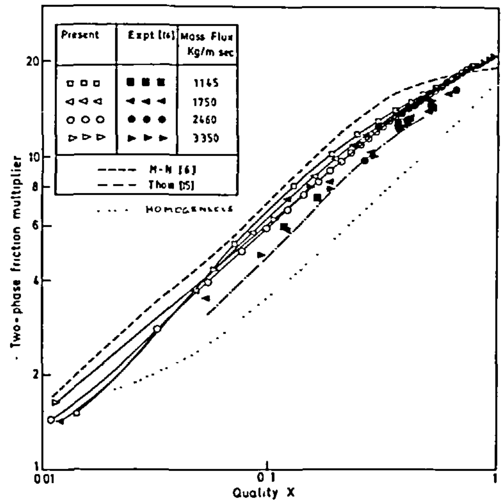

Figure 5 Influence of mass flux on two-phase friction multiplier at 6.89 MPa, diameter = 5 mm

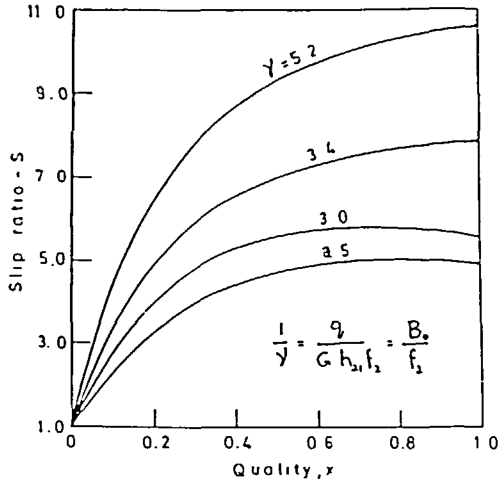

Figure 6 Influence of v on slip ratio

B_0 is the boiling number. The effect of v on the slip ratio is shown in *Figure 6*. The slip ratio remains constant for quality greater than 0.25 at a particular v.

Figure 7 shows the slip factor at mass quality $x = 1.0$ as a function of the pressure. The values calculated by various investigators are also shown for comparison. The slip ratio is greater at lower pressures and less at higher pressures which aspect is observed by other investigators[2]. The agreement with M–N correlation is very good.

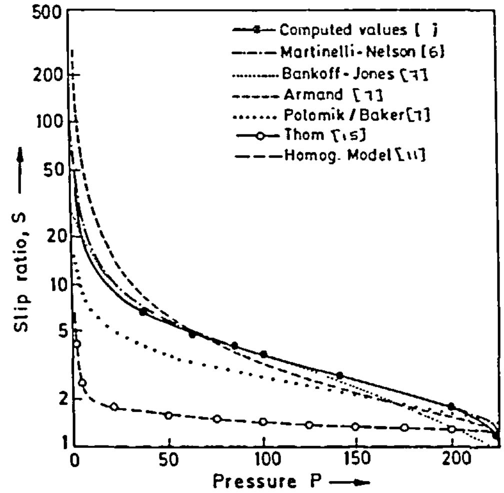

Figure 7 Slip ratio (at $x = 1.0$) variation with system pressure

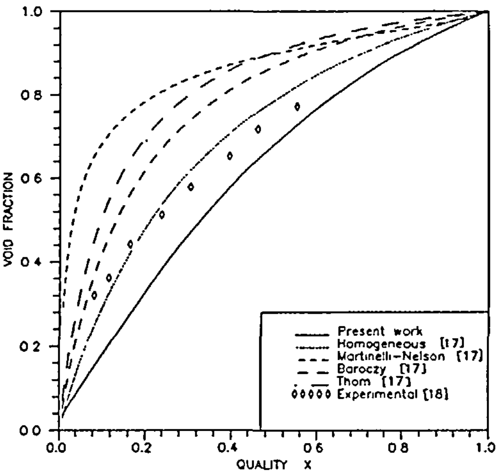

Figure 8 Void fraction variation with quality at 8.27 MPa

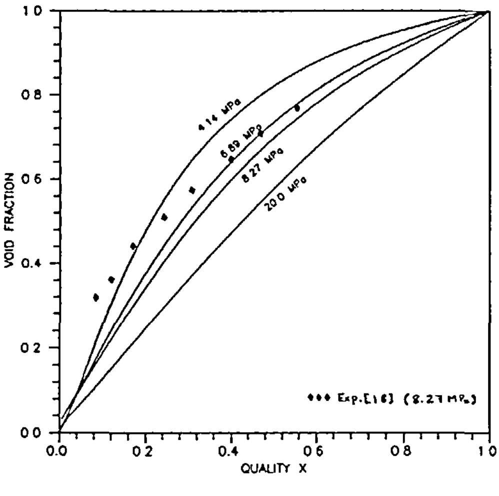

Figure 9 Variation of void fraction with quality at different pressures

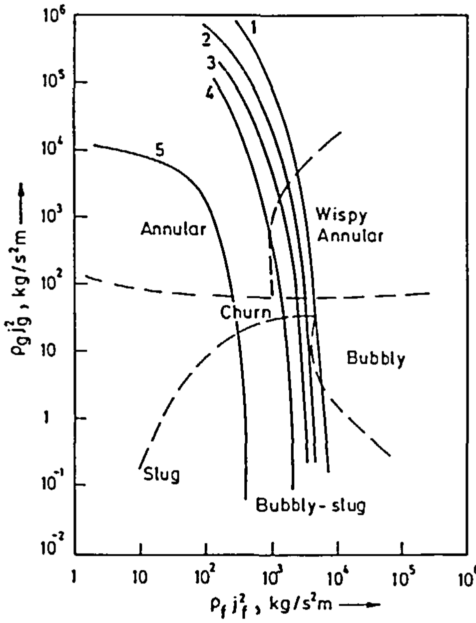

Figure 10 Flow pattern map for vertical upflow

Figure 8 shows the variation of void fraction against quality for a pressure of 8.27 MPa. Various other correlations compiled by Isbin and Biddle[17] are also shown in the Figure along with experimental data of Kelly and Kazimi[18]. It is seen that the model predictions of the present model are reasonably close to the experimental points when compared to the other correlations[17] viz., M–N, Thom, Baroczy and homogeneous model.

Figure 9 shows the plot of void fraction against the mass quality for various pressures. The predicted values compare well with the experimental data[18]. Also, the influence of pressure on the void fraction v/s quality is in good agreement with the accepted trends[11].

Figure 10 shows a plot of flow pattern map for vertical upflow[19]. The different cases discussed above are plotted on the map using the parameters[19]:

$$\rho_1 j_1^2 = \frac{[G(1-x)]^2}{\rho_1} \quad \text{and} \quad \rho_2 j_2^2 = \frac{[Gx]^2}{\rho_2} \tag{27}$$

As can be seen from the *Figure 10*, the test runs cover most of the regimes such as bubbly, slug, churn, wispy annular and annular flow in the map. The regimes covered by a flow situation is dependent upon the pressure and mass flow rate of the flow. The good agreement of the two-phase friction multiplier, slip ratio and void fraction predicted by the present model with experimental data/accepted correlations clearly demonstrates that the present model is capable of predicting the characteristics of flow in most of the flow regimes.

CONCLUSIONS

Based on the physical relationships a model was proposed for computer simulation of steam generator with special emphasis on description of the relevant two-phase flow process. It was shown that it is possible to compute using finite element method the two-phase phenomena, slip ratio and two-phase friction factor directly from the model equations without the help of external correlations, provided one additional momentum balance equation for slip velocity is taken into consideration. The results show a good agreement with the most common correlations and experimental data.

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